Guided Waves Propagating Along the Magnetostatic Field at a Plane Boundary of a Semi-Infinite Magnetoionic Medium

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Abstract—The characteristics of the surface waves supported by a plane boundary of a semi-infinite region of gyrotropic plasma are investigated for the case in which the direction of the magnetostatic field is parallel to both the interface and the propagation direction. Two cases are considered, one for which the plasma is terminated by a perfectly conducting screen, and the other for which it is terminated by a semi-infinite region of free space. Surface waves are found to be propagated for all frequencies below both the plasma and the gyromagnetic frequency in the first case, and below both the plasma and $1/\sqrt{2}$ times the upper hybrid resonant frequency in the second case. The characteristics of the surface waves are discussed, and numerical results of the phase velocity and the propagation coefficient of the surface waves along the interface, as well as their attenuation rates normal to the interface, are given.

I. Introduction

The PROPAGATION of electromagnetic waves in a stratified plasma layer has application both to the ionospheric wave propagation and to the plasma sheath problem connected with the reentry communication. In a recent book, Wait [1] has given a number of examples of electromagnetic wave propagation in stratified plasma media with and without the presence of the external static magnetic field and with emphasis on their application to the wave propagation beneath the ionosphere. One of the simplest examples of stratified plasma media, which has received considerable attention in recent times, is a perfectly conducting plane screen covered with a semi-infinite layer of gyrotropic plasma with the magnetostatic field being parallel to the screen [2]–[4]. The characteristics of the waves guided along the screen in a direction perpendicular to the static magnetic field have been investigated for the simplest case of a homogeneous and loss-free plasma. Wait [5] has considered the extension of the above analysis for the case of an inhomogeneous plasma.

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Another simple configuration of a plane stratified medium which has been subjected to a considerable investigation is a plane interface between two semi-infinite dielectric regions. Epstein [6] has shown that a surface wave can exist along a plane interface between free space and a nonconducting medium with a negative dielectric constant. Tamir and Oliner [7] have treated the wave propagation along a plane interface between an isotropic plasma and free space and, in confirmation of the results of Epstein, have shown that a surface wave can propagate along a vacuum-plasma interface for frequencies below $1/\sqrt{2}$ times the plasma frequency. The nature of the guided waves at a vacuum-magnetoactive plasma interface has also been considered for the case in which the static magnetic field is parallel to the interface, but is perpendicular to the propagation direction [8]–[11].

In view of its possible application to whistler propagation, there is current interest in the study of wave propagation in a plane stratified gyrotropic plasma for the case of propagation in the direction of the external magnetostatic field. In this paper, two simple problems of the above category are studied, namely, the surface wave propagation at a perfectly conducting plane screen covered with magnetoionic medium, and at a plane interface between free space and magnetoionic medium. In each case, the direction of the magnetostatic field is assumed to be parallel to both the screen and the propagation direction.

II. General Considerations

It is convenient to start with the introduction of a rectangular coordinate system x, y, and z. The half-space $z>0$ is assumed to be filled with a homogeneous, loss-free magnetoionic medium which is permeated throughout by a uniform static magnetic field in the x direction. Consequently, the electric and the magnetic vectors are specified by the following time-harmonic Maxwell's equations [12]:

$$\nabla \times \boldsymbol{E} = i\omega\mu_0 \boldsymbol{H}; \qquad \nabla \times \boldsymbol{H} = -i\omega\epsilon_0 \boldsymbol{\varepsilon} \cdot \boldsymbol{E} \tag{1}$$

where μ_0 and ϵ_0 are, respectively, the permeability and the permittivity of free space. A harmonic time dependence of the form $e^{-i\omega t}$ is assumed for all the field components. The components of the relative tensor permittivity $\boldsymbol{\epsilon}$ are given by the following matrix:

$$\boldsymbol{\epsilon} = \begin{bmatrix} \epsilon_3 & 0 & 0 \\ 0 & \epsilon_1 & i\epsilon_2 \\ 0 & -i\epsilon_2 & \epsilon_1 \end{bmatrix} \tag{2}$$

where

$$\epsilon_1 = \frac{\Omega^2 - 1 - R^2}{\Omega^2 - R^2}; \qquad \epsilon_2 = \frac{R}{\Omega(\Omega^2 - R^2)} \tag{3a}$$

$$\epsilon_3 = \frac{\Omega^2 - 1}{\Omega^2}; \qquad \Omega = \frac{\omega}{\omega_p} \qquad R = \frac{\omega_c}{\omega_p} \tag{3b}$$

and ω, ω_p, and ω_c are the source, the electron plasma, and the electron gyromagnetic angular frequencies, respectively.

Since only the two-dimensional problem $(\partial/\partial y = 0)$ is to be considered, the field components may be sought in the following form:

$$\boldsymbol{E}(x, z) = \boldsymbol{E} e^{ik_0\zeta x + ik_0\eta z}; \quad \boldsymbol{H}(x, z) = \sqrt{\frac{\epsilon_0}{\mu_0}}\, \boldsymbol{H} e^{ik_0\zeta x + ik_0\eta z} \tag{4}$$

where k_0 is the propagation coefficient of electromagnetic waves in free space. Using (4) in (1), the following relations are easily derived:

$$E_y = -\frac{1}{\eta} H_x; \qquad E_z = \frac{1}{\epsilon_1 \eta}(-i\epsilon_2 H_x - \epsilon_3 \zeta E_x) \tag{5}$$

$$H_y = \frac{\epsilon_3}{\eta} E_x; \qquad H_z = -\frac{\zeta}{\eta} H_x \tag{6}$$

$$(\epsilon_1\epsilon_3 - \epsilon_1\eta^2 - \epsilon_3\zeta^2) i E_x + \epsilon_2 \zeta H_x = 0 \tag{7}$$

$$-\epsilon_2\epsilon_3\zeta i E_x + (-\epsilon_1^2 + \epsilon_2^2 + \epsilon_1\eta^2 + \epsilon_1\zeta^2) H_x = 0. \tag{8}$$

A nontrivial solution of (7) and (8) easily may be shown to be possible only if

$$\eta^4 + B\eta^2 + C = 0 \tag{9}$$

where

$$B = \frac{1}{\epsilon_1}\left[\zeta^2(\epsilon_1 + \epsilon_3) - (\epsilon_1^2 - \epsilon_2^2 + \epsilon_1\epsilon_3)\right] \tag{10a}$$

$$C = \frac{\epsilon_3}{\epsilon_1}\left[(\zeta^2 - \epsilon_1)^2 - \epsilon_2^2\right]. \tag{10b}$$

With the use of (3a) and (3b), the expressions for B and C, given in (10a) and (10b), may be simplified to yield

$$B = \frac{1}{\Omega^2(\Omega^2 - 1 - R^2)}\left[\{2\Omega^4 - 2\Omega^2(1 + R^2) + R^2\}\zeta^2 - \{2\Omega^4 - 2\Omega^2(2 + R^2) + 2 + R^2\}\right] \tag{11}$$

$$C = \frac{(\Omega^2 - 1)}{\Omega^4(\Omega^2 - 1 - R^2)}\left[\Omega^4(\zeta^2 - 1)^2 + 2\Omega^2(\zeta^2 - 1) - \Omega^2 R^2(\zeta^2 - 1)^2 + 1\right]. \tag{12}$$

Also, it may be proved that

$$B^2 - 4C = \frac{4R^2\zeta^2\Delta}{\Omega^4(\Omega^2 - 1 - R^2)^2};$$

$$\Delta = \Omega^2 - 1 + \frac{R^2(\zeta^2 - 1)^2}{4\zeta^2}. \tag{13}$$

The solutions of (9) are given by

$$\eta_{1,2}^2 = -\frac{B}{2} \pm \frac{R\zeta\sqrt{\Delta}}{\Omega^2(\Omega^2 - 1 - R^2)}. \tag{14}$$

In obtaining η from η^2, a branch-cut along the positive real axis of the η^2 plane is implied. Consider the following four cases: 1) η_1 real, η_2 imaginary; 2) both η_1 and η_2 are complex such that $\eta_2 = -\eta_1^*$; 3) both η_1 and η_2 are purely imaginary; and 4) both η_1 and η_2 are purely real. It is convenient for the following analysis to determine the values of R, Ω, and ζ which correspond to the preceding four cases. This problem was studied by Seshadri [13] in a different connection, and the results are presented in Fig. 1 for $R<1$ and in Fig. 2 for $R>1$.

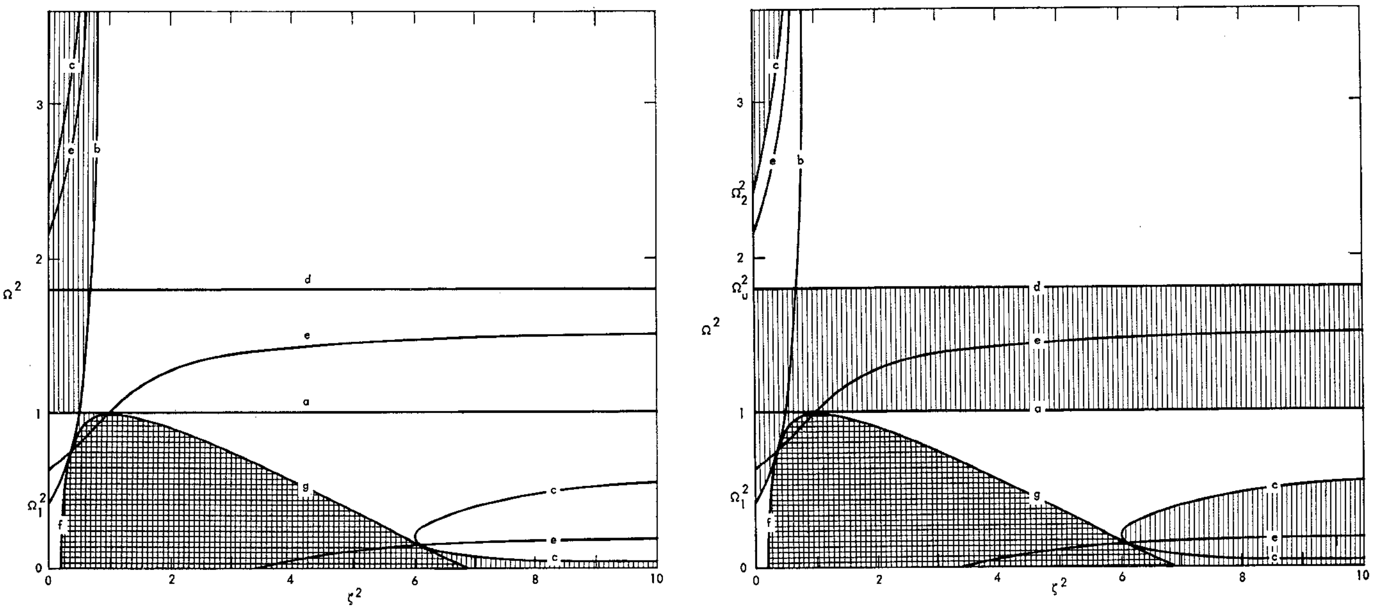
In view of (4) and the foregoing discussion, it is obvious that, in general, any field component, say $E_x(x, z)$, has the following form:

$$E_x(x, z) = \left[E_1 e^{ik_0\eta_a z} + E_2 e^{ik_0\eta_b z}\right] e^{ik_0\zeta x} \qquad z > 0 \tag{15}$$

where

$$\eta_a = \pm\,\eta_1 \qquad \eta_b = \pm\,\eta_2. \tag{16}$$

When η is purely imaginary or complex with a positive imaginary part, the positive sign in (16) has to be chosen since the negative sign will result in the fields growing exponentially for large z. When η is real, either the positive or negative sign in (16) has to be chosen so as to fulfill the radiation condition. For a particular value of R, Ω, and ζ for which η_1 is real, let Ω alone be changed gradually, so that at a particular value of $\Omega = \Omega_p$, η_1 will change from being purely real to being purely imaginary. Therefore, $\Omega = \Omega_p$ corresponds to a branch-point on the real axis of the Ω plane. Along the real axis, η_1 is real on one side of the branch-point and is purely imaginary on the other side. On the purely imaginary side, it is obvious that $\eta_a = \eta_1$. To obtain the correct value of η_a on the other side of the branch-point, it is only necessary to analytically continue around the branch-point in a manner that ensures analyticity in the upper half of the Ω plane. Such a procedure will guarantee, in the case of a transient source, that there is no response before the starting of the source. A detailed treat-



(a) (b)

Fig. 1(a) Regions of propagation and nonpropagation for the mode 1 for $R^2=0.8$.
(b) Regions of propagation and nonpropagation for the mode 2 for $R^2=0.8$.[1]

ment of this method of determining the proper values of $\eta_a(\eta_b)$ when $\eta_1(\eta_2)$ is real is given by Seshadri and Wu [14], and the results alone are summarized in Table I, with particular reference to $R=0.5$ and $R=1.5$ since the numerical results given in this paper pertain to only these two values of R.

With the help of (4) through (7), and (15), the expressions for $E_y(x, z)$, $H_x(x, z)$, and $H_y(x, z)$ easily may be obtained to be given by

$$E_y(x, z) = -i\left[\frac{Z_a}{\eta_a}E_1 e^{ik_0\eta_a z} + \frac{Z_b}{\eta_b}E_2 e^{ik_0\eta_b z}\right]e^{ik_0\zeta x} \tag{17}$$

$$H_x(x, z) = i\sqrt{\frac{\epsilon_0}{\mu_0}}\left[Z_a E_1 e^{ik_0\eta_a z} + Z_b E_2 e^{ik_0\eta_b z}\right]e^{ik_0\zeta x} \tag{18}$$

$$H_y(x, z) = \epsilon_3\sqrt{\frac{\epsilon_0}{\mu_0}}\left[\frac{E_1}{\eta_a}e^{ik_0\eta_a z} + \frac{E_2}{\eta_b}e^{ik_0\eta_b z}\right]e^{ik_0\zeta x} \tag{19}$$

where

$$Z = -\frac{\left[\epsilon_1\epsilon_3 - \epsilon_1\eta^2 - \epsilon_3\zeta^2\right]}{\epsilon_2\zeta}. \tag{20}$$

It is desired first to investigate the nature of the guided waves propagating along the magnetostatic field at a perfectly conducting plane screen with a semi-infinite region of magnetoionic medium.

[1] The following applies to Figs. 1 through 9.

a) $\Omega^2 = 1$

b) $\zeta^2 = \zeta_1^2 = \dfrac{(\Omega - \Omega_1)(\Omega + \Omega_2)}{\Omega(\Omega + R)}$

c) $\zeta^2 = \zeta_2^2 = \dfrac{(\Omega + \Omega_1)(\Omega - \Omega_2)}{\Omega(\Omega - R)}$

d) $\Omega^2 = 1 + R^2$

e) $\zeta^2 = \zeta_4^2 = \dfrac{(\Omega^2 - \Omega_3^2)(\Omega^2 - \Omega_4^2)}{(\Omega^2 - \Omega_5^2)(\Omega^2 - \Omega_6^2)}$

f) $\zeta^2 = \zeta_5^2 = 1 + \dfrac{2}{R^2}\left[(1 - \Omega^2) - \sqrt{(\Omega^2 - 1)(\Omega^2 - 1 - R^2)}\right]$

g) $\zeta^2 = \zeta_6^2 = 1 + \dfrac{2}{R^2}\left[(1 - \Omega^2) + \sqrt{(\Omega^2 - 1)(\Omega^2 - 1 - R^2)}\right]$

$$\Omega_{\substack{1\\2}} = \mp\frac{R}{2} + \sqrt{\left(\frac{R}{2}\right)^2 + 1}$$

$$\Omega_{\substack{3\\4}}^2 = 1 + \frac{R^2}{2} \mp \sqrt{\frac{R^4}{4} + \frac{R^2}{2}}$$

$$\Omega_{\substack{5\\6}}^2 = \frac{1}{2}\left[1 + R^2 \mp \sqrt{1 + R^4}\right].$$

The curve $\zeta^2=\zeta_4^2$ intersects the curves $\zeta^2=\zeta_2^2$ and $\zeta^2=\zeta_1^2$ at $\Omega=\Omega_\alpha$ and $\Omega=\Omega_\beta$, respectively.

(vertical hatching) $= \eta$ real

(cross hatching) $= \eta$ complex ($\eta_1 = -\eta_2^*$)

(blank) $= \eta$ imaginary.

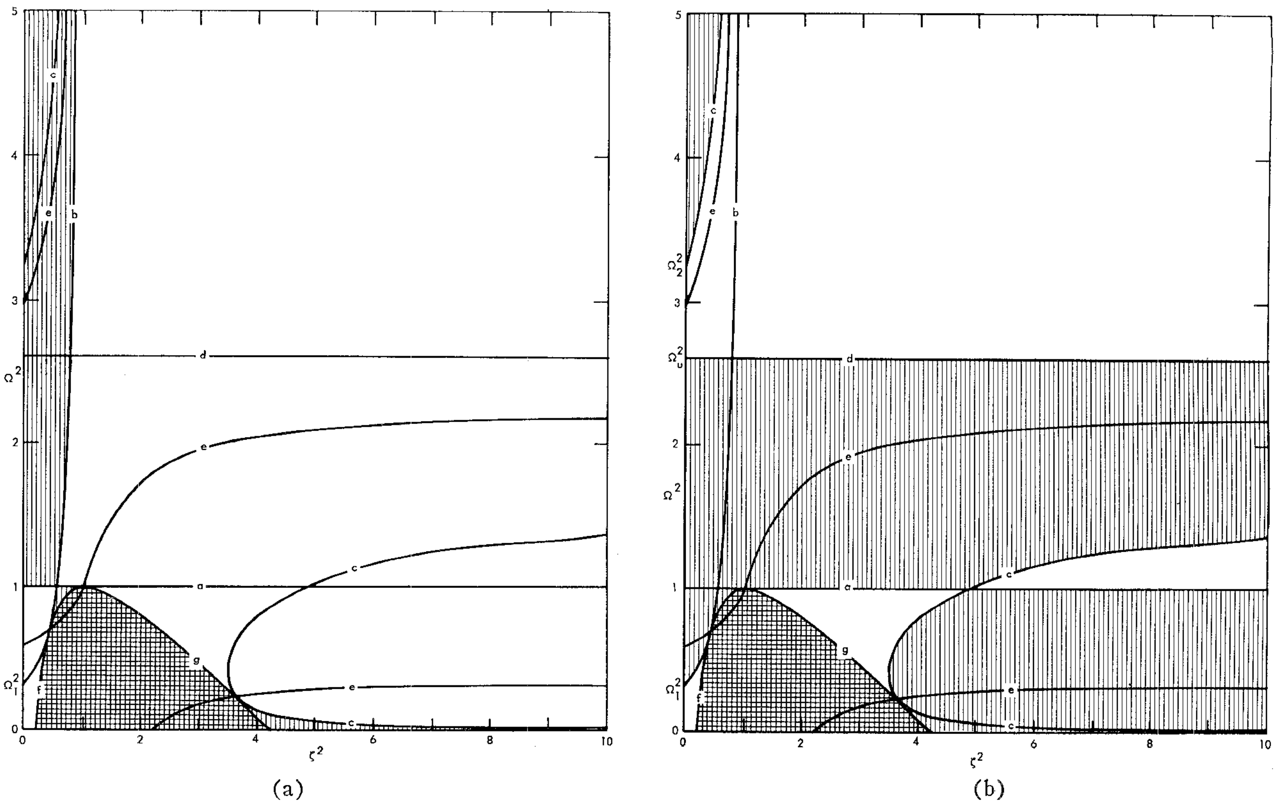


Fig. 2(a) Regions of propagation and nonpropagation for the mode 1 for $R^2=1.6$.
(b) Regions of propagation and nonpropagation for the mode 2 for $R^2=1.6$.

TABLE I
PROPER VALUES OF η_a AND η_b

$0<\Omega<\Omega_\alpha$	$\zeta_6<\zeta<\zeta_2$	$\eta_a=\eta_1$
$\Omega_\beta<\Omega<1$	$\zeta_1<\zeta<\zeta_5$	$\eta_a=-\eta_1$
$1<\Omega<\infty$	$0<\zeta<\zeta_1$	$\eta_a=\eta_1$
$0<\Omega<\Omega_\alpha$	$\zeta_6<\zeta<\infty$	$\eta_b=-\eta_2$
$\Omega_\alpha<\Omega<\min(R, 1)$	$\zeta_2<\zeta<\infty$	
$\Omega_1<\Omega<\Omega_\beta$	$0<\zeta<\zeta_1$	$\eta_b=\eta_2$
$\Omega_\beta<\Omega<1$	$0<\zeta<\zeta_5$	
$1<\Omega<\max(R, 1)$	$0<\zeta<\zeta_2$	
$\max(R, 1)<\Omega<\Omega_u$	$0<\zeta<\infty$	
$\Omega_2<\Omega<\infty$	$0<\zeta<\zeta_2$	$\eta_b=\eta_2$

III. PERFECTLY CONDUCTING SCREEN COVERED WITH A MAGNETOIONIC MEDIUM

The application of the boundary conditions $E_x(x, 0)=E_y(x, 0)=0$, on the perfectly conducting screen, together with (15) and (17), yields the proper dispersion equation (PDE) specifying ζ:

$$\text{PDE:} -\epsilon_1\eta_a\eta_b+\epsilon_3(\zeta^2-\epsilon_1)=0. \tag{21}$$

The improper dispersion equation (IDE) is obtained from (21) by changing $\eta_a\eta_b$ to $-\eta_a\eta_b$. On multiplying PDE and IDE and noting that

$$\eta_a^2\eta_b^2=\eta_1^2\eta_2^2=C;\quad \epsilon_1-\epsilon_3=\epsilon_1^2-\epsilon_2^2-\epsilon_1\epsilon_3, \tag{22}$$

the following possible solutions are obtained for ζ^2:

$$\zeta_{a,b}^2=\left[1\pm(-S)^{-1/2}\right]^{-1} \text{ where } S=\Omega^2-1-R^2. \tag{23}$$

In order to determine the proper solution of (21), it is necessary to ascertain if (23) satisfies (21) by direct substitution. It is easily verified from (23) that ζ_a is real for $0<\Omega<\Omega_u$, where it is less than unity, and ζ_b is real for $0<\Omega<R$, where it is greater than unity. Note that $\Omega_u=\sqrt{1+R^2}$ is called the upper hybrid resonant frequency.

For values of ζ given by (23), (12) reduces to

$$C=-(\Omega^2-1)^2/\Omega^4S. \tag{24}$$

For $0<\Omega<\Omega_u$, $C>0$ and the Case 1, corresponding to η_1 being real and η_2 being imaginary, does not occur.

$$B=B_n \text{ for } \zeta=\zeta_n \text{ and } \Delta_n=\left(\frac{B_n}{2}\right)^2-C;\ n=a, b. \tag{25}$$

Using (11), (12), and (23), it can be shown that

$$B_1=f_1(\Omega)/\Omega^2S \tag{26}$$

$$\Delta_1 = R^2 h_1(\Omega)/4\Omega^4 S^2[1 + \sqrt{-S}] \tag{27}$$

where

$$f_1(\Omega) = R^2/(1 + \sqrt{-S}) + 2(\Omega^2 - 1)\sqrt{-S} \tag{28}$$

and

$$h_1(\Omega) = R^2/(1 + \sqrt{-S}) + 4(\Omega^2 - 1)\sqrt{-S}. \tag{29}$$

It can be shown that $f_1(\Omega_a) = 0$, where $1/\sqrt{2} < \Omega_a < 1$. For $0 < \Omega < \Omega_a$, $f_1(\Omega) < 0$ and, hence, $B_1 > 0$, and for $\Omega_a < \Omega < \Omega_u$, $f_1(\Omega) > 0$ and, hence, $B_1 < 0$. It is also possible to show that $h_1(\Omega_{a0}) = 0$ where $\Omega_{a0} > \Omega_a$ and $\sqrt{3/2} < \Omega_{a0} < 1$. Clearly $h_1(\Omega) < 0$ and, hence, $\Delta_1 < 0$ for $0 < \Omega < \Omega_{a0}$, and $h_1(\Omega) > 0$ and $\Delta_1 > 0$ for $\Omega_{a0} < \Omega < \Omega_u$. Using (23), (21) becomes for $\zeta = \zeta_a$,

$$\frac{1}{\Omega^2(\Omega^2 - R^2)}\left[-\Omega^2 S\eta_a\eta_b + (\Omega^2 - 1)\sqrt{-S}\right] = 0. \tag{30}$$

For $0 < \Omega < \Omega_{a0}$, since $\Delta_1 < 0$, η_1 and η_2 are complex such that $\eta_2 = -\eta_1^*$. Also, since when η is complex, $\eta_a = \eta_1$ and $\eta_b = \eta_2$, it follows that $\eta_a\eta_b < 0$. It can be verified that $\zeta = \zeta_a$ lies in the range $\zeta_1 < \zeta < \zeta_5$ for $(\Omega_\beta <)\Omega_{a0} < \Omega < 1$, and in the range $0 < \zeta < \zeta_1$ for $1 < \Omega < \Omega_u$. Therefore, both η_1 and η_2 are real for $\Omega_{a0} < \Omega < \Omega_u$. From Table I, it follows that for $\Omega_{a0} < \Omega < 1$, $\eta_a = -\eta_1$ and $\eta_b = \eta_2$, and for $1 < \Omega < \Omega_u$, $\eta_a = \eta_1$ and $\eta_b = \eta_2$. Consequently, $\eta_a\eta_b < 0$ for $\Omega_{a0} < \Omega < 1$, and $\eta_a\eta_b > 0$ for $1 < \Omega < \Omega_u$.

With reference to Fig. 3, for $0 < \Omega < 1$, $\eta_a\eta_b < 0$ and, therefore, both the terms inside the square brackets in (30) are negative and hence cannot add to zero. For $1 < \Omega < \Omega_u$, $\eta_a\eta_b > 0$ and, therefore, both the terms inside the brackets in (30) are positive and hence cannot nullify each other. Hence, $\zeta = \zeta_a$ is not a solution of (21).

With the help of (11), (12), and (23), it can be proved that

$$B_2 = f_2(\Omega)/\Omega^2(\Omega^2 - R^2)S \tag{31}$$

$$\Delta_2 = R^2[1 + \sqrt{-S}]h_2(\Omega)/4\Omega^4(\Omega^2 - R^2)^2 S^2 \tag{32}$$

$$f_2(\Omega) = R^2 - \sqrt{-S}\, g(\Omega) \tag{33}$$

$$g(\Omega) = 2\Omega^4 - 2\Omega^2(1 + R^2) + R^2 = 2(\Omega^2 - \Omega_5^2)(\Omega^2 - \Omega_6^2) \tag{34}$$

and

$$h_2(\Omega) = R^2 - \sqrt{-S}\{4\Omega^4 - 4\Omega^2(1 + R^2) + 3R^2\}. \tag{35}$$

It can be deduced that

$$\Omega_5^2 < \min(1, R^2) < \max(1, R^2) < \Omega_6^2 < \Omega_u^2. \tag{36}$$

For $\Omega_5 < \Omega < R$, $g(\Omega) < 0$, and therefore, $f_2(\Omega) > 0$. Then, it follows from (31) and (36) that $B_2 > 0$ for $\Omega_5 < \Omega < R$. From (33), it is found that $f_2(0) < 0$, $f_2(\Omega_5) > 0$, and $df_2/d\Omega^2 > 0$ for $0 < \Omega < \Omega_5$. Therefore, it is obvious that $f_2(\Omega)$ has one zero, say $\Omega = \Omega_b$, in the range $0 < \Omega < \Omega_5$. It may be easily deduced then that $B_2 < 0$ for $0 < \Omega < \Omega_b$, and that $B_2 > 0$ for $\Omega_b < \Omega < R$. Note that $h_2(1) > 0$, $h_2(R) > 0$, and $h_2(\Omega_b) < 0$. It can be shown that $h_2(\Omega)$ has a zero, $\Omega = \Omega_{b0}$, in the range $\Omega_b < \Omega < R$. Consequently, in the range $0 < \Omega < \Omega_{b0}$, $h_2(\Omega)$ and, hence, Δ_2 are both negative, and they are both positive for $\Omega_{b0} < \Omega < R$.

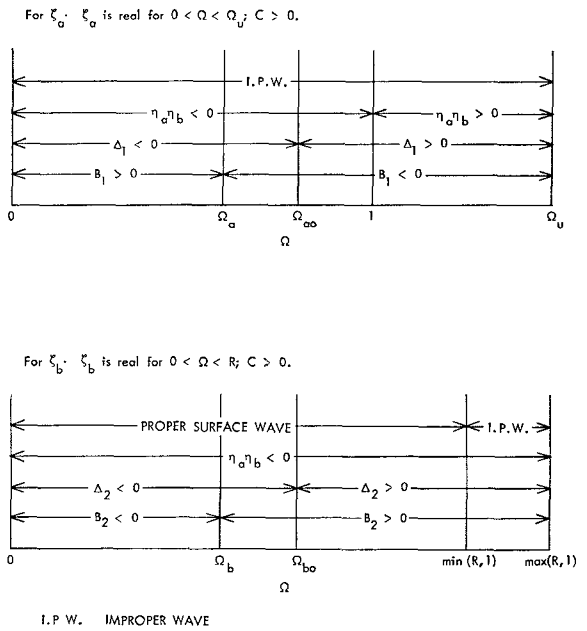


Fig. 3. Frequency ranges of various waves.

Using (23), (21) becomes for $\zeta = \zeta_b$

$$\frac{1}{\Omega^2(\Omega^2 - R^2)}\left[-\Omega^2 S\eta_a\eta_b - (\Omega^2 - 1)\sqrt{-S}\right] = 0. \tag{37}$$

For $0 < \Omega < \Omega_{b0}$, since $\Delta_2 < 0$, both η_1 and η_2 are complex such that $\eta_2 = -\eta_1^*$, and therefore, $\eta_a\eta_b = \eta_1\eta_2 < 0$. For $\Omega_{b0} < \Omega < R$, since $\Delta_2 > 0$ and $B_2 < 0$, both η_1 and η_2 are purely imaginary with the result that $\eta_a = \eta_1$; $\eta_b = \eta_2$ and $\eta_a\eta_b = \eta_1\eta_2 < 0$. With reference to Fig. 3, for $0 < \Omega < R$, since $\eta_a\eta_b < 0$, it follows that the first term inside the brackets in (37) is negative. For $0 < \Omega < \min(R, 1)$, the second term within the brackets in (37) is positive, but for $\min(R, 1) < \Omega < \max(R, 1)$, it is negative. It is then obvious that (37) becomes equal to zero when $\zeta = \zeta_b$ for the frequency range, $0 < \Omega < \min(R, 1)$. Consequently, it may be concluded that $\zeta = \zeta_b$ for $0 < \Omega < \min(R, 1)$ is the only *proper* solution of (21), and that (21) has no other real solutions. Since the imaginary parts of η_1 and η_2 are both positive for $0 < \Omega < \min(R, 1)$, it follows from (15) that for $\zeta = \zeta_b$, the fields propagate without any attenuation along the x direction but decay exponentially in the z direction. Consequently, the wave associated with the propagation coefficient $\zeta = \zeta_b$ is a surface wave.

For $0 < \Omega < \Omega_{b0}$, the surface wave field is easily shown to be the product of two factors, one of which decays

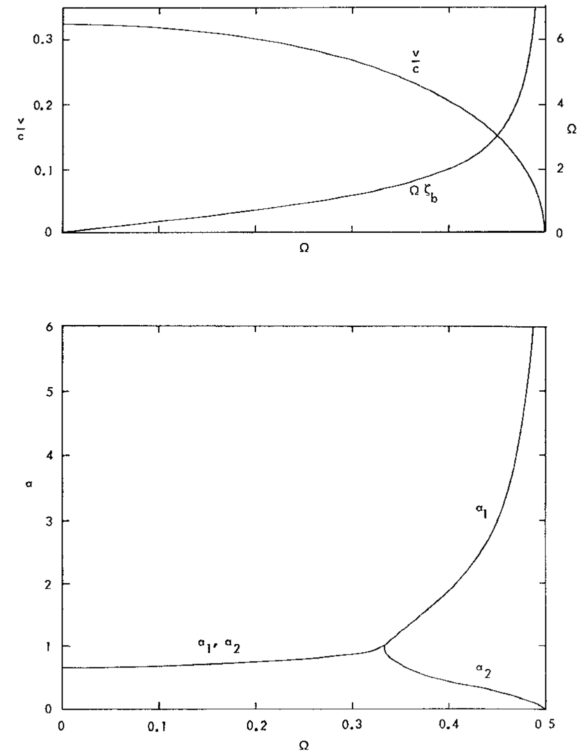


Fig. 4. Characteristics of surface waves along a perfectly conducting screen covered with a magnetoionic medium for $R=0.5$.

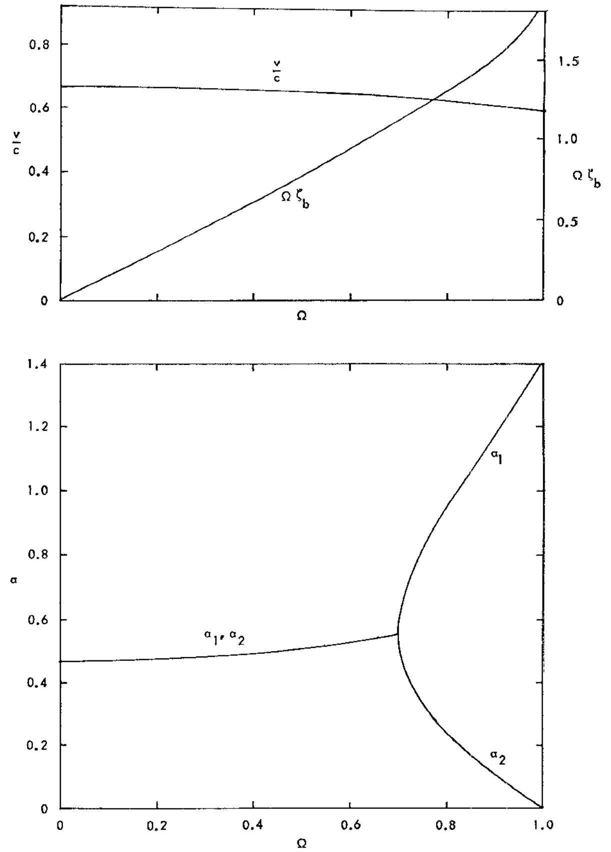


Fig. 5. Characteristics of surface waves along a perfectly conducting screen covered with a magnetoionic medium for $R=1.5$.

exponentially, and the other shows an oscillatory behavior normal to the screen. For $\Omega_{b0}<\Omega<\min(R, 1)$, the surface wave field is the sum of two components, each of which decays exponentially at different rates normal to the screen. From (23), it is obvious that the phase velocity $v=c/\zeta_b$ of the surface waves is always *less than* the free space electromagnetic wave velocity c.

The normalized phase velocity $v/c=1/\zeta_b$ and the normalized propagation coefficient $\Omega\zeta_b$ are depicted in Figs. 4 and 5. The surface waves are seen to be forward waves in the sense that their phase and group velocities are of the same sign and their phase velocity always decreases with the frequency. For $R<1$, the surface waves have a resonance at $\Omega=R$, for which frequency the phase velocity goes to zero. The phase velocity of the surface waves always increases as the strength of the external magnetostatic field is increased. The normalized attenuation rates, which are defined by the relation $\alpha_{1,2}=\Omega \operatorname{Im} \eta_{1,2}$, are also plotted in Figs. 4 and 5 as a function of frequency. As indicated earlier for $0<\Omega<\Omega_{b0}$, $\alpha_1=\alpha_2$, and they increase with the frequency. For $\Omega_{b0}<\Omega<\min(R, 1)$, α_1 and α_2 are different, α_1 increases and α_2 decreases with the frequency. For $\Omega=\min(R, 1)$, $\alpha_2=0$, and for $R<1$, $\alpha_1=\infty$ at the surface wave resonant frequency $\Omega=R$.

Adachi and Mushiake [15], [16] have also investigated this problem previously and have obtained results which are different from those presented here in that they obtain, in addition to the surface waves, nonattenuated plane waves with a propagation coefficient along the perfectly conducting screen given by $\zeta=\zeta_a$, in the frequency range $1<\Omega<\Omega_u$. They state that one of the plane waves is left-handed and is incident on the conducting plane and the other is right-handed and is reflected from the plane. From physical considerations, it is not reasonable to expect a homogeneous plane wave propagating at an angle to the conducting plane for an infinite, open structure. Moreover, the existence of the incoming or the incident plane wave ipso facto violates the outgoing or the radiation condition. From Table I, it is seen that the radiation condition in a magnetoionic medium is the same as that in free space for both the modes in the frequency range $1<\Omega<\Omega_u$. As a result, both the modes should be required to have outwardly traveling phase fronts, in order that the radiation condition be fulfilled. Note that any field component, such as $E_x(x, z)$, has, in general, four linearly independent solutions. Two of the solutions have outwardly traveling phase fronts and, therefore, satisfy the radiation condition in the frequency range $1<\Omega<\Omega_u$. The other two solutions have inward traveling phase fronts and hence violate the radiation condition for $1<\Omega<\Omega_u$. At the outset, the two incoming solutions which violate the radiation condition are to be ruled out and, as in (15), only the two independent solutions which satisfy the radiation condition have to be retained. Therefore, no incoming or incident plane wave can be obtained as a solution, with the result that the plane waves obtained by Adachi and Mushiake do not constitute a proper solution and that the corresponding $\zeta=\zeta_a$ does not satisfy the proper dispersion equation (21) for $1<\Omega<\Omega_u$.

IV. Vacuum-Plasma Interface

Let the half-space $z>0$ be filled, as before, with a homogeneous, loss-free plasma with a static magnetic field in the x direction. Let the remaining half-space $z<0$ consist of free space with the plane interface $z=0$ separating the semi-infinite regions of free space and plasma. In the free space region, the electric and the magnetic fields are specified by (1) with $\epsilon_2=0$ and $\epsilon_3=\epsilon_1=1$. It is convenient to seek the expressions for the field components in the following form:

$$\boldsymbol{E}(x, z) = \boldsymbol{E}e^{ik_0\zeta x+ik_0\eta z}; \quad \boldsymbol{H}(x, z) = \sqrt{\frac{\epsilon_0}{\mu_0}}\,\boldsymbol{H}e^{ik_0\zeta x+ik_0\eta z}. \tag{38}$$

Note that only the case for which the fields are independent of y is to be considered. With the help of (1) and (38), it can be shown that

$$E_y = -\frac{1}{\eta} H_x \qquad E_z = -\frac{\zeta}{\eta} E_x \tag{39}$$

$$H_y = \frac{1}{\eta} E_x \qquad H_z = -\frac{\zeta}{\eta} H_x. \tag{40}$$

With the help of (38) through (40), it is obvious that $E_x(x, z)$, $E_y(x, z)$, $H_x(x, z)$, and $H_y(x, z)$ are given by the following expressions for $z<0$:

$$E_x(x, z) = E_0 e^{ik_0\zeta x-ik_0\eta_0 z} \tag{41}$$

$$E_y(x, z) = i\frac{H_0}{\eta_0} e^{ik_0\zeta x-ik_0\eta_0 z} \tag{42}$$

$$H_x(x, z) = i\sqrt{\frac{\epsilon_0}{\mu_0}}\, H_0 e^{ik_0\zeta x-ik_0\eta_0 z} \tag{43}$$

$$H_y(x, z) = -\sqrt{\frac{\epsilon_0}{\mu_0}}\,\frac{E_0}{\eta_0} e^{ik_0\zeta x-ik_0\eta_0 z} \tag{44}$$

where

$$\begin{aligned}\eta_0 &= \sqrt{1-\zeta^2} && \zeta<1\\ &= +i\sqrt{\zeta^2-1} && \zeta>1.\end{aligned} \tag{45}$$

The fields given by (41) through (44) are seen to satisfy the radiation condition as z tends to $-\infty$. The boundary conditions that the tangential components of the electric and the magnetic fields are continuous across the plane interface $z=0$, together with (15), (17) through (19), and (41) through (44) result in a set of homogeneous equations, whose nontrivial solution may be shown with the help of (20) to yield the following dispersion equation specifying ζ:

$$\eta_0(\epsilon_1-\zeta^2)\epsilon_3(1-\epsilon_3) + \epsilon_1(\eta_a+\eta_b)(\epsilon_3\eta_0^2+\eta_a\eta_b) + \epsilon_1\eta_0[\eta_a\eta_b(1+\epsilon_3)+\epsilon_3(\eta_a^2+\eta_b^2)] = 0. \tag{46}$$

The proper values of η_a and η_b have to be chosen in a manner indicated already.

Consider the following two cases: Case 1) $0<\zeta<1$ (η_0 real), η_1 and η_2 are both real; Case 2) $1<\zeta<\infty$ (η_0 imaginary); η_1 and η_2 are both purely imaginary or η_1 and η_2 are both complex such that $\eta_2=-\eta_1^*$. It can then be shown that (46) is real for Case 1 and purely imaginary for Case 2. For the remaining cases, (46) is complex and, in general, it will not be possible to choose a real $\zeta=\zeta_s$ which will simultaneously nullify both the real and the imaginary parts of (46). Therefore, the real solutions of (46) are possible only in the above-mentioned two cases.

Since (46) is quite complicated, only a numerical solution appears to be possible. Since the dispersion relations in an unbounded plasma have marked differences, depending on whether the plasma frequency is greater than ($R<1$) or less than ($R>1$), the gyromagnetic frequency, and since it is, therefore, reasonable to anticipate that these differences will also influence the nature of the solutions of (46), the numerical solution of (46) was sought for one value of R less than unity, ($R=0.5$), and another value of R which is greater than unity, ($R=1.5$).

For Case 1, since $0<\zeta<1$, it is seen from Figs. 6 and 7 that there are only three regions (indicated by shading with horizontal lines) for which both η_1 and η_2 are real. For the values of Ω and ζ, corresponding to the above-mentioned three regions, the real solution of (46) was investigated, but no real solutions were found. For Case 1, the real solutions $\zeta=\zeta_s$ will correspond to homogeneous plane waves in both the plasma and the free space regions; these plane waves will be outgoing from the interface $z=0$ in both the regions. An infinite, open structure cannot be expected to support such plane waves. Consequently, the absence of real solutions $\zeta=\zeta_s$ corresponding to Case 1 is in accord with physical considerations.

For Case 2, the real solutions $\zeta=\zeta_s$ were sought for frequencies ranging from $\Omega=0$ to $\Omega=10$, and again the search was limited to those values of ζ for which η_0 is purely imaginary and η_1 and η_2 are either both purely imaginary (indicated by shading with vertical lines) or complex such that $\eta_2=-\eta_1^*$ (indicated by shading with crossed lines). For $R<1$, the real solutions $\zeta=\zeta_s$ were

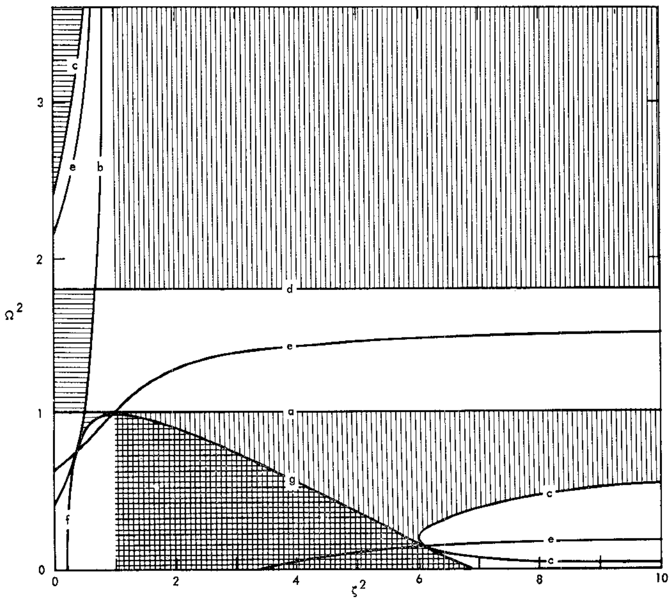


Fig. 6. Possible regions of surface waves for $R^2=0.8$.

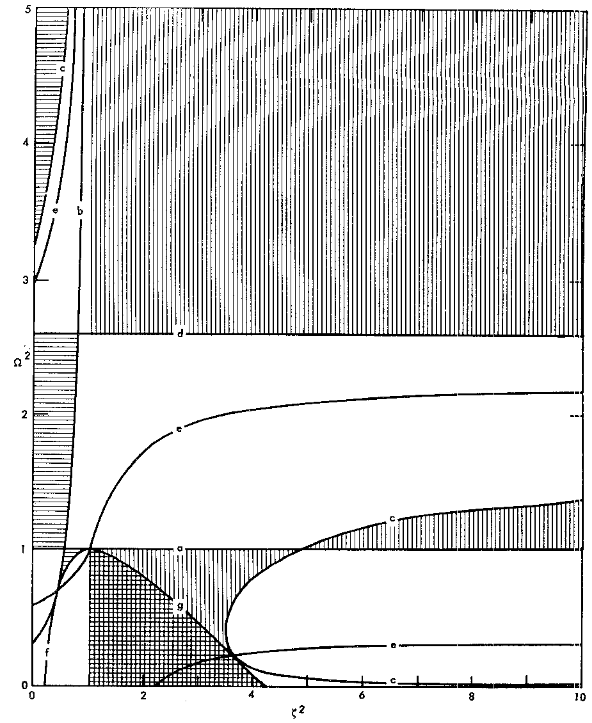


Fig. 7. Possible regions of surface waves for $R^2=1.6$.

found from $\Omega=0$ up to a higher $\Omega=\Omega_m$, which lies between 1 and R. For $R>1$, the real solutions $\zeta=\zeta_s$ were found from $\Omega=0$ to $\Omega=1$. For $R<1$, $\zeta=\zeta_s$ was found to become infinite, and this fact enables the determination of an analytical expression for Ω_m. When ζ tends to infinity, it can be shown with the help of (9) and (10) that

$$\eta_1^2 = -\zeta^2; \qquad \eta_2^2 = -\frac{\epsilon_3}{\epsilon_1}\zeta^2. \tag{47}$$

Using (47), it can be easily shown that for large ζ, (46) becomes

$$-2\epsilon_1\left[1+\sqrt{\frac{\epsilon_3}{\epsilon_1}}\right]\left[\epsilon_3+\sqrt{\frac{\epsilon_3}{\epsilon_1}}\right] = 0. \tag{48}$$

It may be verified easily that for $R<1$, (48) is satisfied if

$$\Omega = \Omega_m = \sqrt{\frac{1+R^2}{2}} = \frac{Q_u}{\sqrt{2}}\,. \tag{49}$$

For $R>1$, it should be noted that (46) can never be satisfied and, therefore, the propagation coefficient for the guided waves never becomes infinite. For $R<1$, the guided waves have a resonance at $\Omega=\Omega_m$, beyond which there is no propagation. The phase velocity of the surface waves vanishes at the resonant frequency $\Omega=\Omega_m$. When the magnetostatic field vanishes, then the surface wave resonant frequency becomes equal to $\Omega=1/\sqrt{2}$, which is the correct value for the case of an isotropic plasma [7].

The normalized phase velocity $v/c=1/\zeta_s$, and the normalized propagation coefficient $\Omega\zeta_s$ of the surface waves are plotted in Figs. 8 and 9. The surface waves are found to be forward waves and their phase velocities approach the free-space electromagnetic wave velocity in the limit of zero frequency. The phase velocity decreases continuously as the frequency is increased and goes to zero at $\Omega=\Omega_m$ for $R<1$. The phase velocity may be shown to attain the value $c\sqrt{(R-1)/R}$ at $\Omega=1$ for $R>1$. The surface waves exist only in the frequency range $0<\Omega<\min(\Omega_m, 1)$. The phase velocity of the surface waves always increases as the strength of the magnetostatic field is increased.

The attenuation rate α_0 of the surface waves normal to the interface, in the free space region, is seen from Figs. 8 and 9 to increase with the frequency, and for $R<1$, α_0 becomes infinite at the surface wave resonant frequency $\Omega=\Omega_m$. In the plasma region, the surface wave field is the product of two factors for Ω less than a critical frequency Ω_c; one of the factors decreases normal to the interface at a rate $\alpha=\alpha_1=\alpha_2$, and the other factor shows an oscillatory behavior. For $\Omega_c<\Omega<\min(\Omega_m, 1)$, the surface wave field is the sum of two components, each of which decays exponentially at different rates α_1 and α_2 normal to the interface. For $R<1$ and for $\Omega>\Omega_c$, both α_1 and α_2 increase rapidly with the frequency and become infinite at the surface wave resonant frequency $\Omega=\Omega_m$. For $R>1$ and $\Omega>\Omega_c$, α_1 increases and α_2 decreases rapidly with the frequency. For $\Omega=1$, beyond which there are no surface waves, $\alpha_2=0$, and α_1 is large, but finite.

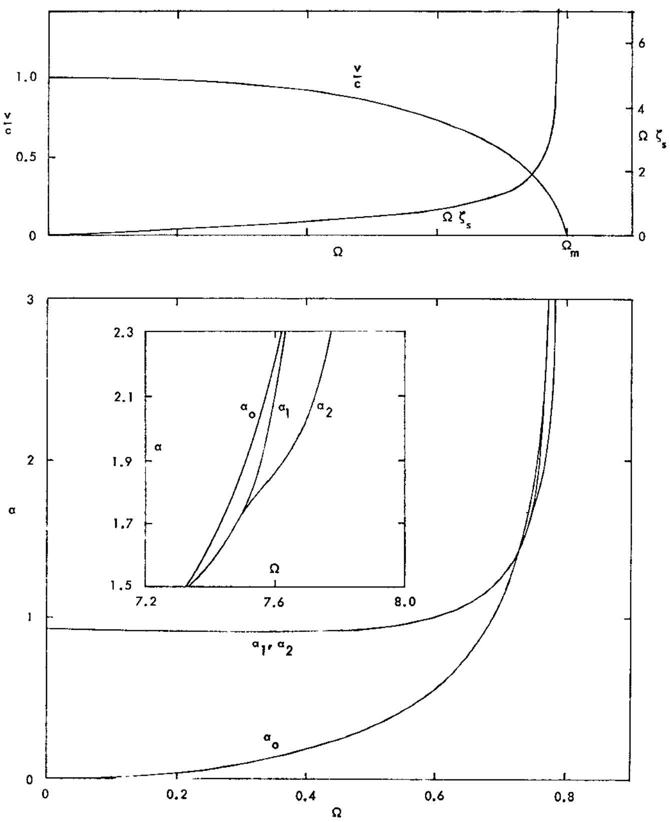


Fig. 8. Characteristics of surface waves along a vacuum-plasma interface for $R=0.5$.

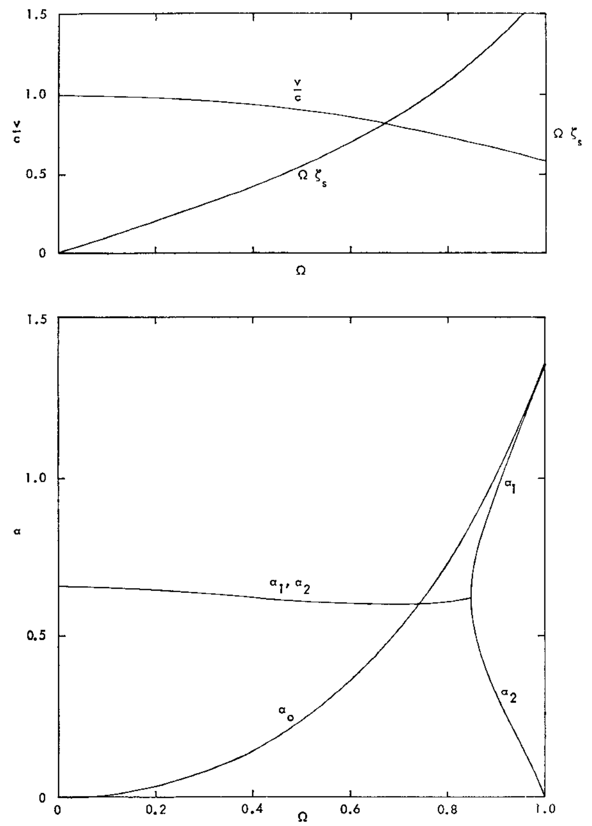


Fig. 9. Characteristics of surface waves along a vacuum-plasma interface for $R=1.5$.

V. Conclusion

The characteristics of the guided waves supported by a plane boundary of a semi-infinite region of magnetoionic medium are studied for the case in which the magnetostatic field is parallel to both the interface and the propagation direction. Two problems are considered, one for which the magnetoionic medium is terminated by a perfectly conducting screen, and the other for which it is terminated by a half-space of vacuum. Surface waves are found to be propagated for all frequencies below both the plasma and the gyromagnetic frequency in the first case, and below both the plasma and $1/\sqrt{2}$ times the upper hybrid resonant frequency in the second case. The characteristics of the surface waves are examined, and numerical results for some typical parameters of interest are given.

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